Additional background material on the Nobel Prize in Physics 1998

The Royal Swedish Academy of Sciences has decided to award **the 1998 Nobel Prize in Physics** jointly to

Professor **ROBERT B. LAUGHLIN**, Stanford University, Stanford, CA, USA

Professor **HORST L. STÖRMER**, Columbia University, New York, NY and Bell Labs, Lucent Technologies, Murray Hill, NJ, USA

Professor **DANIEL C. TSUI**, Princeton University, Princeton, NJ, USA

for their discovery of a new form of quantum fluid with fractionally charged excitations.

This additional background material is written mainly for physicists.

The experimental discovery of the fractional quantum Hall effect, by Horst L. Störmer and Daniel C. Tsui, implied the existence of a previously unknown kind of collective behaviour of electrons. It showed that the physics of electrons in strong magnetic fields is far richer than anyone could have anticipated. The subsequent theoretical explanation of the phenomenon by Robert B. Laughlin, in terms of a new type of quantum fluid, started a major and still active trend in theoretical condensed matter physics. It has had implications also for other fields. The work of Laughlin, Störmer and Tsui has lead to a breakthrough in our understanding of macroscopic quantum effects and to the emergence of an extremely rich set of phenomena with deep and truly fundamental theoretical implications.

Historical background

The Hall effect was discovered already in 1879 by the American Edwin Hall, then a graduate student at Johns Hopkins University in Maryland. He found that a voltage – the Hall voltage – appears across a thin sheet of a conducting material when an electrical current is sent along the sheet in the presence of a perpendicular magnetic field. Normally, the Hall voltage as well as the Hall resistance (the ratio between Hall voltage and current) is proportional to the magnetic field strength. Since it is also proportional to the density of charge carriers, the Hall effect offers a convenient method for measuring charge carrier densities in various materials. The method is routinely used in physics laboratories today.

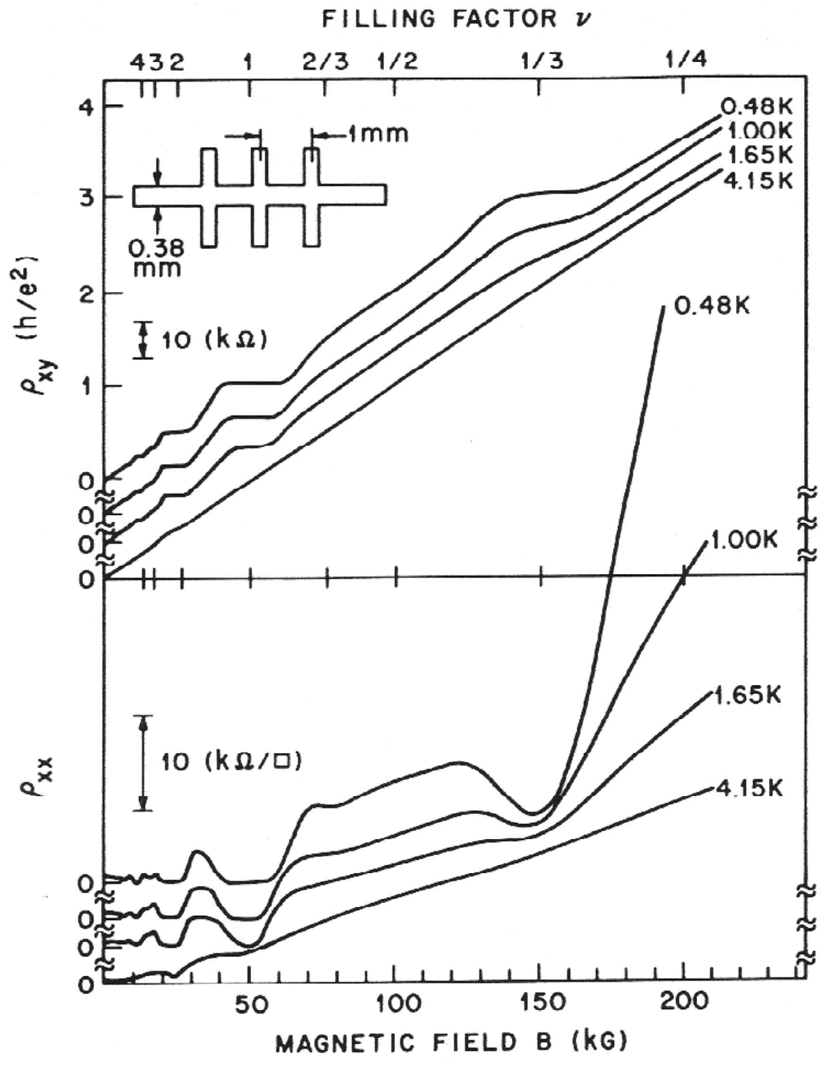
The laws of quantum physics alter the Hall effect significantly at low temperatures if the charge carriers are confined to moving in a plane, for instance along a two-dimensional internal surface of a layered semiconductor material. Through recent remarkable technological achievements, such structures with sufficiently pure and well defined interfaces of essentially atomic thickness and showing these quantum effects, can now be fabricated.

What we now call the *integer quantum Hall effect* was discovered by Klaus von Klitzing in the beginning of 1980 and rendered him the Nobel Prize in Physics 1985. His effect shows up as plateaus in the Hall resistance traced as a function of magnetic field (or particle density) with resistance values extremely close to $(h/e^2)/f$. Here the so called filling factor, f, is an integer number, while e and h are fundamental constants of nature – the elementary charge of the electron and Planck's constant. The filling factor is determined by the electron density and the magnetic flux density. It can most easily be defined as the ratio $f=N/N_{\Phi}$ between the number of electrons N and the number of magnetic flux quanta $N_{\Phi}=\Phi/\Phi_0$. Here Φ is the magnetic flux through the plane and $\Phi_0=h/e$ $=4.1 \cdot 10^{-15}$ Vs is the magnetic flux quantum (a tiny amount of flux indeed; the earth's weak magnetic field of 0.03 millitesla corresponds to almost a million flux quanta per cm^2). When f is an integer the electrons completely fill a corresponding number of the degenerate energy levels (Landau levels) formed in a two-dimensional electron gas under the influence of a magnetic field. Energy dissipation is in this situation associated with excitations over an energy gap typically corresponding to a temperature of 100 kelvin. If the filling factor is a fraction, $f=1/3$ for instance, there is no energy gap in the independent electron model that defines the Landau levels. The gap of a few kelvin that is important for the *fractional quantum Hall effect*, to be described below, is caused by a strongly correlated motion of electrons and is induced by the magnetic field and the repulsive Coulomb interaction between the electrons.

Discovery of an anomalous quantum Hall effect

A couple of years after the discovery of the integer effect, Horst L. Störmer and Daniel C. Tsui of AT&T Bell Laboratories at Murray Hill, New Jersey (now part of Lucent Technologies), were studying the Hall effect using very high quality gallium arsenide-based samples provided by A. C. Gossard, now at the University of California at Santa Barbara. The purity of the samples was so high that the electrons could move ballistically, i.e. without scattering against impurity atoms, over comparatively long distances. To achieve this, the semiconductor sample had to be "modulation"-doped with doping atoms in another layer than the one where the conduction takes place. The molecular beam epitaxy technique that Gossard used so successfully had been developed by A. Cho and others. The large scattering lengths can be achieved at low temperatures and, thus, the experiments had to be performed at or below 1 kelvin and at very high magnetic field strengths. Fields of up to about 20 tesla, i.e. close to one million times the earths's magnetic field, were used in the original experiments. The experiments discovered Hall plateaus at high magnetic field strengths corresponding to a fractional value of the filling factor f. In their first publication they demonstrated a plateau at $f=1/3$. They also found some indications of a plateau at $2/3$, which – by particle-hole symmetry in the lowest Landau level – can be viewed as corresponding to a $1/3$ filling factor for holes.

The discovery of this “anomalous“ quantum Hall effect took the condensed matter community completely by surprise. No-one had anticipated that a fractionally filled Landau level had any particularly interesting properties. Störmer and Tsui were fully aware of the fact that, in contrast to the integer effect, their fractional quantum Hall effect could not be explained within a model ignoring the interactions between electrons. They supposed that the arguments used to understand the integer effect were not applicable. Still, they observed that if they for some reason were applicable anyway, they implied the existence of quasiparticles carrying fractional charge, for instance *e/3* if *f=1/3*.



Graph showing the results of the original experiment where the fractional quantum Hall effect was discovered for filling factor *f=1/3*, around which the Hall resistance has a plateau at low temperatures. In later experiments well defined plateaus appeared also at many other filling factors, accompanied by much sharper minima in the Ohmic resistance than shown for the *1/3* case in the lower panel here (see figure in the press release at http://www.nobel.se/ announcement-98/physics98.html).

Laughlin’s wavefunction - a theorist’s *tour-de-force*

The astonishing discovery of the fractional quantum Hall effect seemed completely at odds with what was understood about the integer effect. At the time it posed a real challenge to theorists. Little progress was made until Robert B. Laughlin, then at Bell Labs, came up with a totally unexpected theoretical explanation. It has turned out to be the corner stone for much of the subsequent developments with important implications also outside this field. Laughlin showed that the electron system condenses into a new type of quantum liquid when its density corresponds to “simple“ fractional filling factors of the form $f=1/m$, where m is an odd integer; $f=1/3$ or $1/5$ for example. He even proposed an explicit many-electron wave function for describing the ground state of this quantum liquid of interacting electrons. This is remarkable since in previously discovered macroscopic quantum phenomena, such as superconductivity, a microscopic understanding on the wave function level was reached only after a long period of steady progress gained by phenomenological approaches. Laughlin also showed that an energy gap separates the excited states from the ground state and that they contain “quasiparticles” of fractional charge $\pm e/m$. This implies that the Hall resistance becomes exactly quantized to m times h/e^2.

To further discuss Laughlin’s novel ideas it is convenient to denote the position (x_j,y_j) of the j:th electron in a two-dimensional plane by a complex number $z_j= x_j- iy_j$. Then, aside from an unimportant exponential factor, the wave function for N electrons can be written as a simple product over all differences between particle positions $(z_j- z_k)$,

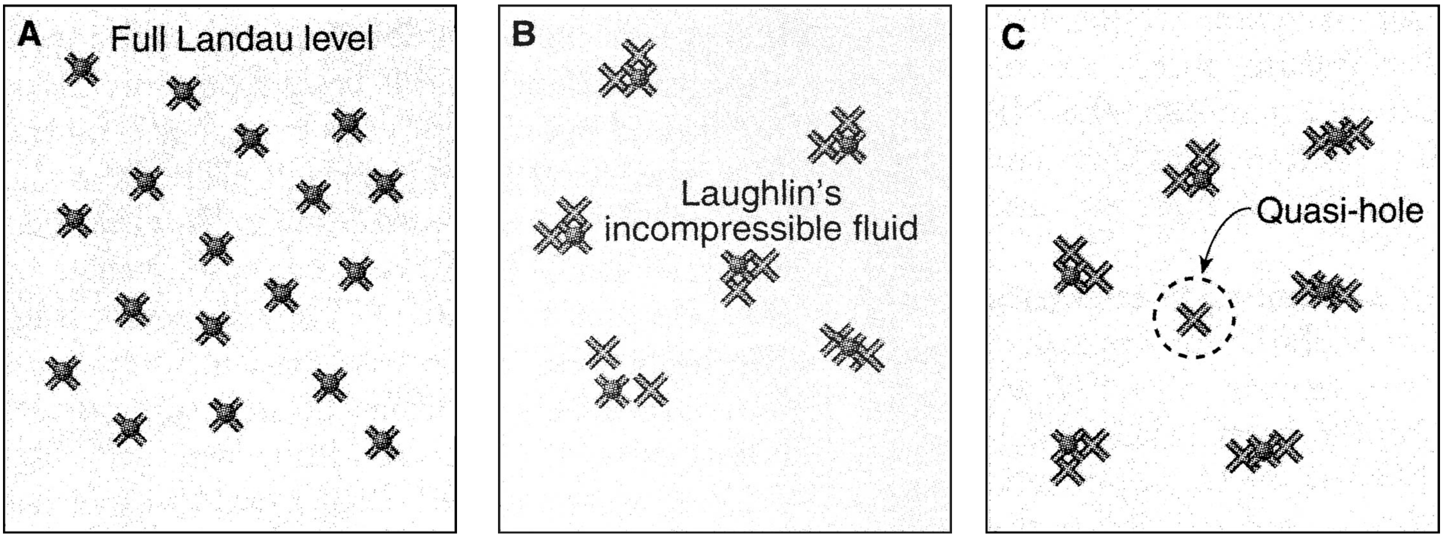
$$\Psi_m(z_1,z_2,z_3,\ldots,z_N) = (z_1- z_2)^m\ (z_1- z_3)^m\ (z_2- z_3)^m \ldots (z_j- z_k)^m \ldots (z_{N-1}- z_N)^m\ . \quad (1)$$

Laughlin arrived at this form of the wave function guided by the theory of ^{4}He, which involves correlated many-particle wave functions (Jastrow functions). Starting from a variational Ansatz, he was able to identify a number of constraints and eliminate most of the variational freedom except for the value of m. It turns out, however, that this parameter can be determined by minimizing the ground state energy corresponding to the trial wave function (1). Laughlin discovered a useful and beautiful analogy between the interacting electrons of the fractional quantum Hall system and a one-component classical plasma of particles interacting with a logarithmic potential. This classical system is well known and, although certainly not trivial, a number of numerical schemes are available for calculating its energy. By using a plasma analogy, Laughlin was therefore able to calculate the energy of his proposed ground state. He found that it could be minimized by choosing the parameter m to be related to the electron density, and hence, the filling factor as $f=1/m$. He also recognized that m has to be an odd integer to ensure that the wave function changes sign upon the exchange of two electrons, $z_2 \times z_3$ for instance, since this is required for particles like electrons which obey Fermi-Dirac statistics (fermions).

Several physicists supported Laughlin’s ideas by demonstrating that his wave function is an excellent approximation to the exact ground state wave function. Exact numerical calculations by Duncan Haldane and his collaborators were particularily important in this respect. Since the complexity of such calculations increases very rapidly with the number of electrons, Haldane could only study systems of a few electrons. Still, by putting them on a sphere he was able to minimize boundary effects and show that Laughlin's wave function, when adapted to the same geometry, accounted for all but a few per cent of the exact wave function.

Fractionally charged quasiparticle excitations

Determining the ground state is only one of the two key ingredients of a description of a physical system. Most properties of the system are governed by low-energy excitations, or by states whose energies are slightly higher than the energy of the ground state. In his seminal 1983 paper Laughlin showed that the low-energy excitations at filling factor $f=1/m$ are rather special: in addition to being separated from the ground state by a finite energy gap they contain quasiparticles carrying fractional charge $\pm e/m$.



Views of three many-electron states following MacDonald (see under Further reading). In these sketches crosses represent zeros of the many-electron wave functions as a function of one coordinate, z_1 say, and dots represent the positions of the other electrons. (A) A full Landau-level state in which the number of zeros equals the number of electrons ($f=1$). (B) A condensed $f=1/3$ quantum fluid state in which three zeroes are bound to the positions of each electron. (C) A state with a single zero, that is, a single quasi-particle excitation which is bound to a maximum (if hole-like) or minimum (if electron-like) in the disorder potential rather than to other electrons.

In this context it is useful to recall Bertrand Halperin's early interpretation of the significance of Laughlin's wave function. By using Laughlin's plasma analogy mentioned above, Halperin concluded that the many-electron wave function must have a finite density of zeros equal to the density of magnetic flux quanta in the external magnetic field. With this picture in mind, one can consider the N interacting electrons described by Laughlin's wave function (1), freeze the positions of particles number *2* through N, and view Ψ_m as a single-electron wave function for the "representative" particle number one. One finds immediately that this particle indeed does see a finite density of points where the wave function goes to zero. In fact each particle sees m zeros (or vortices) located at the positions of the other particles. One may say that there are m zeros (or vortices) *bound* to each particle. This binding constitutes a fundamental "ordering" in the fractional quantum Hall effect and plays the role of an order parameter. We shall briefly return to this point below.

Laughlin argued that the elementary excitations from his ground state amounts to creating extra vortices. Imagine, for instance, that we remove an electron (of integral charge) from the system. The m vortices "left over" then unbind in Laughlin's picture, each "quasiparticle" carrying a charge of minus $1/m$:th of the integral charge that was removed. Similarily, if an ordinary electron is added to Laughlin's liquid it is immediately split up into an odd number of quasiparticles, each carrying the same fraction of the electron's charge. Since the electrons are optimally correlated in the ground state, reducing the repulsive Coulomb interaction to a minimum, an addition or a subtraction of a single electron or flux quantum disturbs this order at a considerable energy cost. For this reason, the $f=1/m$ quantum states represent *condensed* many-particle ground states. Since the electron positions are not fixed, as in a solid, the Laughlin state is a new type of quantum liquid. It follows from the arguments above that this liquid can be compressed only at the expense of the creation of quasiparticles, which costs energy. This is why Laughlin's quantum liquid is said to be incompressible, although, of course, this is strictly true only if the liquid is squeezed gently enough.

Direct experimental verifications of energy gap and charge fragmentation

The existence of the fractional quantum Hall effect itself, *i.e.* of plateaus in the Hall resistance around filling factors like $1/3$, is an indirect verification of the theory outlined. However, the central elements of the theory have also been directly verified by experiments: *viz.* there is a gap in the excitation spectrum and excited states contain localized quasiparticle excitations of fractional charge. Before we proceed to describe these experiments, we pause to contemplate the crucial fact that the Hall plateaus have a finite width around the fractional filling factor $1/m$ (otherwise, the fractional quantum Hall effect could, of course, not have been observed). The finite-width plateaus can be understood as being due to a trapping of the first quasiparticles created as one goes away from $1/m$ filling by, say, changing the magnetic field. The quasiparticles are trapped due to the residual disorder present even in very clean materials. Being trapped, the quasiparticles can neither move nor dissipate energy. Larger changes in magnetic field overwhelm the trapping capacity and the plateaus disappear.

Let us first consider the direct experimental verification of the energy gap. At a finite temperature, quasiparticles can be created in pairs carrying charge $+e/m$ (electron-like) and $-e/m$ (hole-like) while maintaining overall charge neutrality. The quasiparticles will be mobile, dissipate energy and hence contribute to the ordinary resistance of the system. In analogy with the situation in a superconductor, or an insulator, the energy gap Δ for creating pairs is the sum of the energies it takes to create an electron-like and a hole-like quasiparticle. The experimental value of Δ can be obtained from the temperature dependence of the Ohmic resistance, which is of activated type. Early experiments, by the group of Hiroyaki Sakaki in Japan and Klaus von Klitzing in Germany, and the Bell Labs group, only allowed a qualitative comparison with theory. This is because the samples were not pure enough. Disorder tends to suppress the fractional effect, whereas it enhances the integer effect. In 1989 R. L. Willett and J. H. English of AT&T Bell Labs in collaboration with Störmer, Tsui, and Gossard had access to better samples. Their experimental values of Δ, 5-10 kelvin or 0.5-1 meV depending on sample, agree with Laughlin's predictions (when trivially modified, mainly to account for the finite thickness of the two-dimensional electron layer) to within about 20%.

In addition to quasiparticle excitations, the new quantum liquid has also collective excitations in the form of density (and spin density) fluctuations. These can be characterized by a wave vector $\boldsymbol{k}$ and may in the long wavelength limit $k \to 0$ be thought of as a coherent superposition of quasiparticle excitations. In the short wavelength limit $k \to \infty$ the density fluctuations represent an incoherent quasiparticle excitation. Steven Girvin and Allan MacDonald of Indiana University, together with Philip Platzman of Bell Labs have developed a theory for these collective excitations in an analogy with Feynman's theory of superfluid helium. The theory, which builds on Laughlin's description of the ground state, predicts a finite gap in the excitation spectrum. The value of this gap at $k=0$ was measured in 1993 for the $f=1/3$ state by Aron Pinczuk and his collaborators at Bell Labs, who used inelastic light scattering. The agreement with theory was good. Incidentally, the gap has a minimum at finite wave vector k_0 in full analogy with the Landau/Bijl/ Feynman "roton minimum". According to the theory the magnitude diminishes as m increases and the gap disappears at $m=7$ or 9, signalling an instability of the Laughlin electron liquid with respect to the creation of an electron solid – a Wigner lattice – with lattice constant $1/ k_0$. Such phase transitions have been observed experimentally.

The second central element in the theoretical explanation of the fractional quantum Hall effect is the fragmentation of charge. Direct verification of the existence of fractionally charged quasiparticles have so far been obtained by three groups using two different methods: by Vladimir Goldman and B. Su of the State University of New York at Stony Brook in 1995 from measurements of resonant tunneling currents and in 1997 by groups lead by Mordehai Heiblum of the Weizmann Institute of Science in Israel and by Christian Glattli of the French Atomic Energy Commision in France. These two groups both measured the shot noise in tunneling currents, which clearly showed that the current was carried by objects with charge $e/3$.

The shot noise measurements were made with a very high level of precision and represent a remarkable achievement. However, the theory of shot noise is long since established and is well understood, which means that the results of these experiments are rather easily interpreted. In the limit of zero temperature, for instance, the shot noise is proportional to the current and to the charge carried by the flowing particles. At finite temperatures the shot noise is modified in a well known manner. The parameters needed to compare with theory, such as the sample temperature, can be determined by independent measurements leaving the quasiparticle charge as the only undetermined parameter. Fitting to the theory gives the quasiparticle charge as $e/3$ with an accuracy of the order of 10%.

Fermions, Bosons or … Anyons?

In many cases progress in understanding the fractional quantum Hall effect has been guided by analogies with another dissipationless "superfluid", *viz.* helium. ^{4}He is a liquid of bosonic particles and can Bose-Einstein condense into a macroscopic superfluid quantum state. Electrons and the quasiparticles in the fractional quantum Hall systems are fermions, or are they? It turns out that the statistics in two dimensions is ambiguous. Consider a gauge transformation in which one attaches magnetic flux tubes containing m flux quanta to each of the electrons. The dynamics of the electrons remain unaffected because they never see any actual magnetic field, which exists only in the places they cannot reach (on the other electrons). But imagine exchanging two particles by slowly moving them around each other. The quantum probability amplitude for the system to return to its original state will contain the usual statistics phase plus an additional term, the Berry phase, which is just what would occur in the Aharonov-Bohm effect when a charge circles a flux tube. The extra phase is $m\pi$. If m were continuous, the statistics would interpolate continuously between Fermi-Dirac and Bose-Einstein and we would have a physical realization of the "fractional statistics" discussed by the Norwegian physicists J. M. Leinaas and J.Myrheim; instead of fermions or bosons we would have F. Wilczek's "anyons".

If m is an odd integer, and if we change the underlying ("bare") statistics of the electrons from Fermi-Dirac to Bose-Einstein to compensate for the flux tubes, the physics of the system remains unchanged. Hence one can describe the physics of the fractional quantum Hall effect by hard-core "composite" bosons which carry (fake) magnetic flux (hard-core bosons because they cannot be allowed to be at the same place simultaneously). The mean-field theory of these objects replace the actual local flux density by the mean density. Since there are m physical flux quanta per electron at filling factor $f=1/m$, we have a perfect cancellation between the physical flux and the average "statistical" flux. What has happened is that in changing from Fermi-Dirac to Bose-Einstein statistics, the electrons have "swallowed up" the external magnetic field (on the average).

In this new “dual” boson picture a Landau-Ginzburg theory of the fractional quantum Hall effect was formulated by Girvin and MacDonald in analogy with the corresponding theory for superfluid helium, where the particles are bosons. The approximate theory correctly describes the important order of the fractional quantum Hall system, which in this picture is a subtle type of off-diagonal long-range order. As we recall, in the original picture the order is related to the binding of the wave function zeros (vortices) to the electrons. In the dual picture there is a relation to the Laughlin wave function, which is well understood and therefore one can argue that we have a good understanding of the “meaning” of the Laughlin wave function (1) and of the fractional quantum Hall effect itself, in the sense that we have a mean field theory that captures the essential physics

New surprises keep emerging in a still very active field

The physics of fractional quantum Hall systems is an active field both experimentally and theoretically. In the first few years following the original disovery a large number of quantum Hall plateaus were found as better, higher-purity samples became available. The additional plateaus correspond to more complicated fractional filling factors $f=p/q$, where p is an even or odd integer and q is an odd integer. The new plateaus were explained independently by Haldane, Laughlin, and Halperin in terms of a “hierarchy” af fractional quantum states with Laughlin’s $1/m$-states as “parent” states. An alternative generalization of the theory of the $1/m$-states by J. Jain from 1989 is particularily interesting; Jain describes the *fractional* quantum Hall effect as the *integer* effetc for composite particles where an even number of flux quanta are bound to each electron (composite fermions).

In 1989 it was discovered that when the magnetic field is tuned so that the Hall resistance equals the resistance quantum divided by 1/2 or 1/4, rather than by 1/3 or 1/5, new phenomena emerge. These “even-denominator“ quantum liquids are fermi liquids fundamentally different from the “odd-denominator“ ones, which further demonstrates the rich physics of electrons in strong magnetic fields.

In the last few years much interest has also focused on the role played by the edges of fractional quantum Hall devices. The gapless edge excitations of a fractional quantum Hall fluid are “chiral” Luttinger liquids. Chiral, beacuse in contrast to the electrons in real one-dimensional wires, they move only in one direction without being backscattered. This gives a very beautiful set of connections with conformal field theory and all the activity there in recent years. Most recently the role played by the spin of the electron in quantum Hall magnets has been the topic of active study.

In conclusion, the experimental discovery of the fractional quantum Hall effect and its theoretical explanation in terms of a new incompressible quantum liquid with fractionally charged excitations has lead to a breakthrough in our understanding of macroscopic quantum phenomena and to the emergence of an extremely rich set of phenomena with deep and truly fundamental theoretical implications, such as the fractionalization of the electron’s charge.

Further reading

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